Malaysian Journal of Mathematical Sciences 16(4): 673–681 (2022)
https://doi.org/10.47836/mjms.16.4.02

Malaysian Journal of Mathematical Sciences

Journal homepage: https://mjms.upm.edu.my

# The Order Product Prime Probability and Commutativity Degree for Some Finite Groups

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*Received: 28 May 2020*
*Accepted: 18 July 2022*

## Abstract

Statistical approach often provide techniques in exploring more on the algebraic properties of finite groups. In this paper, two new group probabilities are defined in which the group elements having certain common characteristic are considered. Let $G$ be a finite group, the order product prime probability is defined as the probability that two randomly selected elements $x$ and $y$ in $G$ satisfy $|x||y| = p^s$ and the order product prime commutativity degree is the probability that elements satisfying the above property commutes. Some formulas for computing these probabilities for dihedral groups are obtained.

**Keywords:** dihedral groups; order product prime probability; order product prime commutativity degree.

## 1 Introduction

The idea of group probability was first introduced by [10]. The probability that a pair of elements in a finite group $G$ commute is called as the commutativity degree of the group and denoted as $P(G)$. After Miller's work, many researchers have developed interest and various achievements has been made, for instance [5] worked on symmetric groups for their commutativity degrees. [6] used a different way to compute the commutativity degree of $G$ where he found that the commutativity degree of $G$ is equal to the number of conjugacy classes divides the order of $G$. By some calculations, [6] and [9] obtained the maximum value of the commutativity degree of finite nonabelian groups which is $P(G) \leq \frac{5}{8}$. Besides that, many other results on the upper and lower bounds for various probability have been found, for example [16] have obtained the lower bound of the commutativity degree of groups. [8] studied the probability that the commutator of two group elements is equal to a given element which is called the $g$-commutativity degree.

More recently, [12] obtained the exact value of the commutativity degree of the generalized quaternion groups, dihedral groups, semidihedral groups and quasi dihedral groups. More researches related to commutativity degree of groups and their extension can be found in [[11], [15], [14], [13], [4], [7]]. The previous concepts of the mention researches are strictly associated with the conception of commutativity degree of a group. On the other side, in 2018, [1] introduced the coprime probability of a group where it is defined as the probability of a random pair of elements in a group $G$ is coprime which is the greatest common divisor of the order of $x$ and $y$ in $G$ is equal to one. The authors found the probability for $p$-groups and for some dihedral groups. Later in 2019, Zulkifli and Mohd Ali in [21] and [20] made an extensive research on the coprime probability in which the scope of the group is on the nonabelian metabelian groups of order less than 24 and order 24, respectively.

Besides the probability of groups, there were many researches have been done in exploring the dihedral groups. These includes the researches by [19], [18] and [17].

However, there are no studies on the probabilities for finite groups by considering the relationship between the product of the order of the elements and a prime power. Accordingly, two new notions called the order product prime probability and the order product prime commutativity degree of groups are introduced in this research. Groups that are considered in this research only the finite dihedral groups and $p$-groups where $p$ is prime.

## 2 Notations and Preliminaries

Some basic concepts, notations and preliminaries result that are needed in this research are given in this section starting with the definition of dihedral groups given by [3].

**Definition 2.1.** *For $n \geq 3$, the $n$-th dihedral group is defined as a group consists of rigid motions of a regular $n$-gon, denoted by $D_n$. The dihedral groups, $D_n$ of order $2n$ (or degree $n$) can be presented in a form of generators and relations given as follows:*

$$D_n = \langle a, b \,|\, a^n = b^2 = e, ba = a^{-1}b \rangle.$$

Firstly, remarks that all groups considered in this research are finite and the investigation covered all dihedral groups of certain degrees. In this research, the identity of a group $G$ is denoted by $e$, the cyclic groups of order $n$ by $\mathbb{Z}_n$, the order of a group $G$ by $|G|$ and the order of an element $x$ in $G$ by $|x|$. Furthermore, the result on the commutativity degree of dihedral group $D_n$ which is given by [2] as in Theorem 2.1 is needed for the following section is stated in this section.

**Theorem 2.1.** *Let $D_n$ be the dihedral group of degree $n$. Then the commutativity degree of $D_n$ is $P(D_n) = \dfrac{n+3}{2|D_n|}$ if $n$ is odd and $P(D_n) = \dfrac{n+6}{2|D_n|}$ if $n$ is even.*

## 3 Order Product Prime Probability

In this section, a new definition which is the order product prime probability of a group is given. Later, some results on dihedral groups and $p$-groups are given in general.

**Definition 3.1.** *Let G be a finite group. The probability that two randomly selected elements $x, y \in G$ satisfy $|x||y| = p^s$ for some prime p that divides $|G|$ where $s$ is a non-negative integer, is define as*

$$P_{op(p)}(G) = \frac{\left|\{(x,y) \in G \times G \mid |x||y| = p^s\}\right|}{|G|^2}.$$

**Example 3.1.** *Consider $D_3$ as the dihedral group of order six that is $D_3 = \{e, a, a^2, b, ab, a^2b\}$ where $|e| = 1$, $|a| = |a^2| = 3$, and $|b| = |ab| = |a^2b| = 2$. Here, 2 and 3 are the prime divisors of $|D_3|$. Next,*

$$\left|\{(x,y) \in D_3 \times D_3 \mid |x||y| = 2^s\}\right| = \left|\{e, b, ab, a^2b\} \times \{e, b, ab, a^2b\}\right|$$

$$\left|\{(x,y) \in D_3 \times D_3 \mid |x||y| = 3^s\}\right| = \left|\{e, a, a^2\} \times \{e, a, a^2\}\right|.$$

*Therefore, $P_{op(2)}(D_3) = \dfrac{4^2}{4 \cdot 9} = \dfrac{4}{9}$ and $P_{op(3)}(D_3) = \dfrac{3^2}{4 \cdot 9} = \dfrac{1}{4}$.*

Next, in Theorem 3.1, Theorem 3.2, Theorem 3.3 and Theorem 3.4, the probability that two randomly selected elements $x, y$ in a group, satisfy $|x||y| = 2^s$, where $s$ is non-negative integer for dihedral groups are given.

**Theorem 3.1.** *Let $D_n$ be a dihedral group such that $n = q^m$, $q \neq 2$, for some positive integer $m$ and prime number $q$, then $P_{op(2)}(D_n) = \dfrac{(n+1)^2}{4n^2}$ and $P_{op(q)}(D_n) = \dfrac{1}{4}$.*

**Proof**: Let $D_n = \langle a, b \mid a^n = b^2 = e, ba = a^{-1}b\rangle$ where $n = q^m$, $q \neq 2$ where $|D_n| = 2q^m$. Then, 2 divide $|D_n|$. Notice that $|b| = |a^kb| = 2$, $1 \leq k \leq n-1$, and 2 does not divide $q^m$ for any $m$ since $q \neq 2$. Moreover, $\left|\langle a\rangle\right| = n = q^m$, so each of the rotations has order a power of $q$, this implies that non of nontrivial rotations can pair off with any of the reflections so that the product of their orders gives $2^s$ where $s$ is non-negative integer. Thus the pairs of elements in $D_n \times D_n$ whose orders product yields $2^s$ are exactly members of the collection $\{e, b, a^kb\} \times \{e, b, a^kb\}$, where $1 \leq k \leq n-1$. Therefore,

$$\left|\{(x,y) \in D_n \times D_n \mid |x||y| = 2^s\}\right| = \left|\{e, b, a^kb\} \times \{e, b, a^kb\}\right| = (n+1)^2.$$

Thus, the order product prime probability of $D_n$ with respect to 2 is $P_{op(2)}(D_n) = \dfrac{(n+1)^2}{4n^2}$.

Another prime divisor of $|D_n|$ is $q$. In this case, by using similar argument as in the previous case, only the elements in the subgroup $\langle a\rangle$ served as the elements satisfy the condition $|x||y| = q^s$ since $|b| = |a^k b| = 2$, for $1 \le k \le n-1$ and 2 does not divide $q^s$. Thus the pairs of elements in $D_n \times D_n$ whose orders product yields $p^s$ are exactly members of the collection $\langle a\rangle \times \langle a\rangle$. Therefore, $\Big|\big\{(x,y) \in D_n \times D_n \,\big|\, |x||y| = p^s\Big| = \Big|\langle a\rangle \times \langle a\rangle\big\}\Big| = n^2$. Therefore, the order product prime probability of $D_n$ with respect to $q$ is $P_{op(q)}(D_n) = \dfrac{n^2}{4n^2} = \dfrac{1}{4}$.

There is no other prime factor of $|D_n|$ different from 2 and $q$. Thus, the proof is complete.

**Theorem 3.2.** *Let $D_n$ be a dihedral group such that $n = q_1q_2$ where $q_1$ and $q_2$ are distinct primes different from 2, then $P_{op(2)}(D_n) = \dfrac{(n+1)^2}{4n^2}$, $P_{op(q_1)}(D_n) = \dfrac{q_1^2}{4n^2}$ and $P_{op(q_2)}(D_n) = \dfrac{q_2^2}{4n^2}$.*

**Proof**: Let $D_n = \langle a, b \mid a^n = b^2 = e, ba = a^{-1}b\rangle$ where $n = q_1q_2$, $q_1$ and $q_2$ are distinct primes different from 2. Then the prime divisors of $|D_n|$ are 2, $q_1$ and $q_2$. Here, each reflection has order 2 and by Lagrange's theorem all nontrivial rotations have order either $q_1$, $q_2$ or $q_1q_2$. However non of $q_1$ or $q_2$ or $q_1q_2$ can be expressed as a power of 2, then from Theorem 3.1, it follows that $P_{op(2)}(D_n) = \dfrac{(n+1)^2}{4n^2}$.

Next, for $q_1$, neither $q_2$ nor $q_1q_2$ can be written as powers of $q_1$. Next, the rotation elements of order $q_1$ need to be determined. In fact $|a^r| = q_1$ if and only if $(r, q_1) = 1$. Note that: $(r, n) \ne 1$, for if $(r, n) = 1$, then $|a^r| = q_1q_2$. Since $(r, n) \ne 1$, $(r, q_1) = 1$ and $r \le n-1$ then $r = kq_2$, $k = 1, 2, \ldots, q_1 - 1$. So the elements whose order product gives powers of $q_1$ are exactly members of the subset $S = \{e, a^{kq_2}\}$ where $k = 1, 2, \ldots, q_1 - 1$. Therefore,

$$\Big|\big\{(x,y) \in D_n \times D_n \,\big|\, |x||y| = q_1^s\big\}\Big| = |S \times S| = (q_1 - 1 + 1)^2 = q_1^2,$$

and hence $P_{op(q_1)}(D_n) = \dfrac{q_1^2}{4n^2}$. Similarly, $P_{op(q_2)}(D_n) = \dfrac{q_2^2}{4n^2}$.

**Theorem 3.3.** *Let $D_n$ be a dihedral group with $n = 2^k$, for some positive integer $k$, then $P_{op(2)}(D_n) = 1$.*

**Proof**: Let $D_n = \langle a, b \mid a^n = b^2 = e, ba = a^{-1}b\rangle$ where $n = 2^k$ where $k \in \mathbb{N}$. Thus $|D_n| = 2^{k+1}$. Then by Lagrange's theorem and the fact that 2 is a prime number, each element in $D_n$ must be of order $2^m$ for some $0 \le m \le k$. Thus for each $x, y \in D_n \times D_n$ there exists $m, l$ such that $|x| = 2^m$ and $|y| = 2^l$, $0 \le m, l \le k$. Thus, $|x||y| = 2^{m+l}$, for each $(x, y) \in D_n \times D_n$. Therefore, $\Big|\big\{(x,y) \in D_n \times D_n \big|\, |x||y| = 2^s\big\}\Big| = |D_n \times D_n| = |D_n|^2$ and hence,

$$P_{op(2)}(D_n) = \frac{|D_n|^2}{|G|^2} = 1.$$

**Theorem 3.4.** *Let $D_n$ be a dihedral group such that $n = 2q$ where $q$ is a prime number and $q \neq 2$, then $P_{op(2)}(D_n) = \dfrac{n^2+4n+4}{4n^2}$ and $P_{op(q)}(D_n) = \dfrac{1}{16}$.*

**Proof**: Let $D_n = \langle a, b \mid a^n = b^2 = e, ba = a^{-1}b\rangle$. Then $|D_n| = 4q$ and 2 divides $|D_n|$. First, notice that among the rotations, $a^q$ has order 2. Therefore $e$, $a^q$, $b$ and $a^k b$ for $1 \leq k \leq n-1$, are the elements, forming a total of $(n+2)^2$ pairs, that satisfies $|x||y| = 2^s$. Thus,

$$P_{op(2)}(D_n) = \frac{(n+2)^2}{(2n)^2} = \frac{n^2+4n+4}{4n^2}.$$

Another cases in this theorem is, $q$ divide $|D_n|$ where it is similar to the second case of Theorem 2 and hence $P_{op(q)}(D_n) = \dfrac{q^2}{4n^2} = \dfrac{q^2}{4(2q)^2} = \dfrac{1}{16}$.

Next, in Theorem 3.5, the sufficient and necessary condition for which the probability, $P_{op(p)}(G)$ is equal to 1 is given.

**Theorem 3.5.** *$G$ is a finite $p$-group if and only if $P_{op(p)}(G) = 1$.*

**Proof**: ($\Longrightarrow$) Suppose that $G$ is a finite $p$-group. Since the order of any element in $G$ must divide $|G| = p^m$ (Lagrange's theorem) and that $p$ is a prime number, then any pair of elements $(x, y) \in G \times G$ satisfies $|x||y| = p^s$. Therefore, $P_{op(p)}(G) = \dfrac{|G|^2}{|G|^2} = 1$.

($\Longleftarrow$) Suppose $P_{op(p)}(G) = 1$, then the ratio $\dfrac{|\{(x,y) \in G \times G||x||y| = p^s\}|}{|G|^2} = 1$, thus

$$\left|\left\{(x,y) \in G \times G \middle| \, |x||y| = p^s\right\}\right| = |G|^2.$$

That is all pairs of elements $(x, y) \in G \times G$ satisfies $|x||y| = p^s$. Taking $y = e$ gives $|x| = p^s$, $\forall$ $x \in G$, hence $G$ is a $p$-group.

In the previous section, results on the order product prime probability were given. Meanwhile, as an extension to this probability, another new notion which is the order product prime commutativity degree is defined and some results of the order product prime commutativity degree are given in the following section.

## 4 Order Product Prime Commutativity Degree

This section is begin with a new definition of order product prime commutativity degree followed by an example of it.

**Definition 4.1.** *Let $G$ be a finite group. Assume that a pair of elements $x, y \in G$ satisfy $|x||y| = p^s$ for some prime $p$ and $s$ is a non-negative integer, then the order prime commutativity degree of $G$ is given as follows:*

$$P_{o(p)c}(G) = \frac{\Big|\{(x,y) \in G \times G\big|\ |x||y| = p^s,\ xy = yx\}\Big|}{\Big|\{(x,y) \in G \times G\big|\ |x||y| = p^s\}\Big|}.$$

**Example 4.1.** *Consider $D_3 = \{e, a, a^2, b, ab, a^2b\}$ where $|e| = 1$, $|a| = |a^2| = 3$ and $|b| = |ab| = |a^2b| = 2$. Here, if $p = 2$, then $e, b, ab$ and $a^2b$ satisfy $|x||y| = 2^s$, so there will be a total of 16 pairs that is*

$$\Big|\{(x,y) \in D_3 \times D_3\big||x||y| = 2^s\}\Big| = 16.$$

*The commuting pairs are $(e,e)$, $(b,b)$, $(ab,ab)$, $(a^2b,a^2b)$, $(e,b)$, $(e,ab)$, $(e,a^2b)$, $(b,e)$, $(ab,e)$ and $(a^2b,e)$. Therefore, $P_{o(2)c}(D_3) = \dfrac{10}{16} = \dfrac{5}{8}$.*

*If $p = 3$, then $e, a$, and $a^2$ as the elements that obeys the condition $|x||y| = 3^s$, forming total of 10 pairs that commutes, thus $P_{o(3)c}(D_3) = \dfrac{10}{10} = 1$.*

Next, in Theorem 4.1, Theorem 4.2, Theorem 4.3 and Theorem 4.4, the order product prime commutativity degree for dihedral groups are given.

**Theorem 4.1.** *Suppose that $D_n$ is a dihedral group of degree $n = 2^k$ for some positive integer $k$, then $P_{o(2)c}(D_n) = \dfrac{n+6}{4n}$.*

**Proof:** By using a similar argument as in the proof of Theorem 3.3,

$$\Big|\{(x,y) \in D_n \times D_n\big|\ |x||y| = 2^s\}\Big| = \Big|\{(x,y) \in D_n \times D_n\}\Big| = |D_n|^2 = (2n)^2 = 4n^2.$$

As a result and by using Theorem 2.1,

$$\Big|\{(x,y) \in D_n \times D_n\big||x||y| = 2^s, xy = yx\}\Big| = |D_n|^2 P(D_n) = n^2 + 6n.$$

Therefore, $P_{o(2)c}(D_n) = \dfrac{\Big|\{(x,y) \in D_n \times D_n\big|\ |x||y| = 2^s, xy = yx\}\Big|}{\Big|\{(x,y) \in D_n \times D_n\big|\ |x||y| = 2^s\}\Big|} = \dfrac{n+6}{4n}$.

**Theorem 4.2.** *Let $D_n$ be a dihedral group where $n$ is odd integers, then $P_{o(2)c}(D_n) = \dfrac{3n+1}{(n+1)^2}$.*

**Proof:** Let $D_n$ be a dihedral group where $n$ is positive odd integers. Since 2 does not divide $n$ then elements satisfying $|x||y| = 2^s$ are identity element and those elements whose order is 2, that is,

$e$ and $a^{k-1}b$ for $1 \leq k \leq n$ (which is $n+1$ elements all together). These $n+1$ elements forming $(n+1)^2$ pairs altogether. The elements $\{a^{k-1}b : 1 \leq k \leq n\}$ commutes with themselves making $n$ pairs of commuting elements. The elements also commute with the identity element making $2n$ pairs of them. Together with $(e, e)$, we have $n + 2n + 1 = 3n + 1$ pairs altogether of commuting elements. Therefore, by definition we get

$$P_{o(2)c}(D_n) = \frac{\Big|\big\{(x,y) \in D_n \times D_n \big| |x||y| = 2^s, xy = yx\big\}\Big|}{\Big|\big\{(x,y) \in D_n \times D_n \big| |x||y| = 2^s\big\}\Big|} = \frac{3n+1}{(n+1)^2}.$$

**Theorem 4.3.** *Let $D_n$ be a dihedral group of degree $n = 2p$ where $p \neq 2$, then $P_{o(2)c}(D_n) = \dfrac{6n+4}{(n+2)^2}$ and $P_{o(p)c}(D_n) = 1$.*

**Proof:**
Let $D_n$ be a dihedral group where $p \neq 2$. For $p = 2$. Notice that $a^{\frac{n}{2}} \in Z(D_n)$ and has order 2. Therefore $\{e, a^{\frac{n}{2}}, b, a^k b : 1 \leq k \leq n-1\}$ are the elements forming a total of $(n+2)^2$ pairs satisfying $|x||y| = 2^s$. Out of these $e$ commute with all making total pairs $2(n+1)+1$, also $n+1$ commute with themselves and $3n$ more pairs commutes.

$$\begin{aligned} P_{o(2)c}(D_n) &= \frac{\Big|\big\{(x,y) \in D_n \times D_n \big| \; |x||y| = 2^s, xy = yx\big\}\Big|}{\Big|\big\{(x,y) \in D_n \times D_n \big| \; |x||y| = 2^s\big\}\Big|} \\ &= \frac{2n+3+n+1+3n}{(n+2)^2} \\ &= \frac{6n+4}{(n+2)^2}. \end{aligned}$$

For any $p \neq 2$, pairs of elements that satisfy $|x||y| = p^s$ must come from $\langle a \rangle$ and hence $P_{o(p)c}(D_n) = 1$ follows immediately.

Next, in Theorem 4.4, the sufficient and necessary condition for which $P_{o(q)c}(G) = P(G)$ is given.

**Theorem 4.4.** *If $G$ is a $p$-group such that $|G| = q^m$, where $q$ is prime then $P_{o(q)c}(G) = P(G)$.*

**Proof:**
Since the order of each element in a group divides the order of the group, then each element in $G$ has the order $p^i$ for some $0 \leq i < m$. Thus, for any pair $x, y \in G \times G$ there exists $i, j$ such that $|x| = p^i$ and $|y| = p^j$, $0 \leq i, j < m$ if and only if $|x||y| = p^{i+j}$. All pairs of elements in $G \times G$ satisfy condition $|x||y| = q^s$, thus $\Big|\big\{(x,y) \in G \times G\big| \; |x||y| = q^s\big\}\Big| = \Big|\big\{(x,y) \in G \times G\big\}\Big| = |G|^2 = q^{2m}$ and

$$P_{o(q)c}(G) = \frac{\Big|\big\{(x,y) \in G \times G\big| \; |x||y| = q^s, xy = yx\big\}\Big|}{\Big|\big\{(x,y) \in G \times G\big| \; |x||y| = q^s\big\}\Big|} = \frac{q^{2m}P(G)}{q^{2m}} = P(G).$$

## 5 Conclusion

In this paper, two new group probabilities, one is the product of the order of the group elements with prime power and the other is the product of the order of the commuting elements and a prime power are defined. The probabilities are then determined for dihedral groups with certain degrees.

**Acknowledgement** The authors would like to express their gratitude to Universiti Teknologi Malaysia (UTM) for the financial support provided by the UTM Encouragement Research Grant (20J85).

**Conflicts of Interest** The authors declare that no conflict of interests occurs.

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